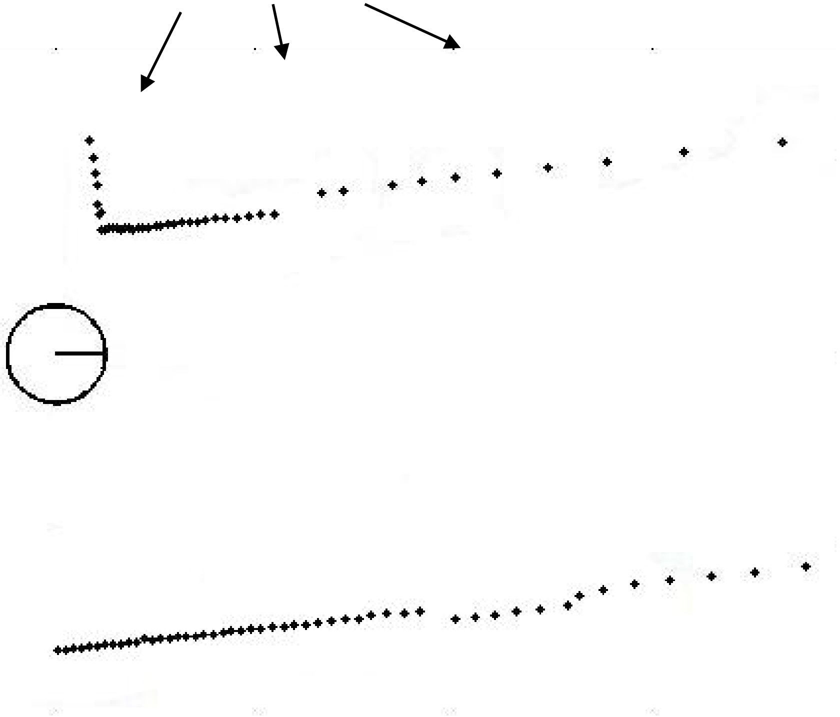
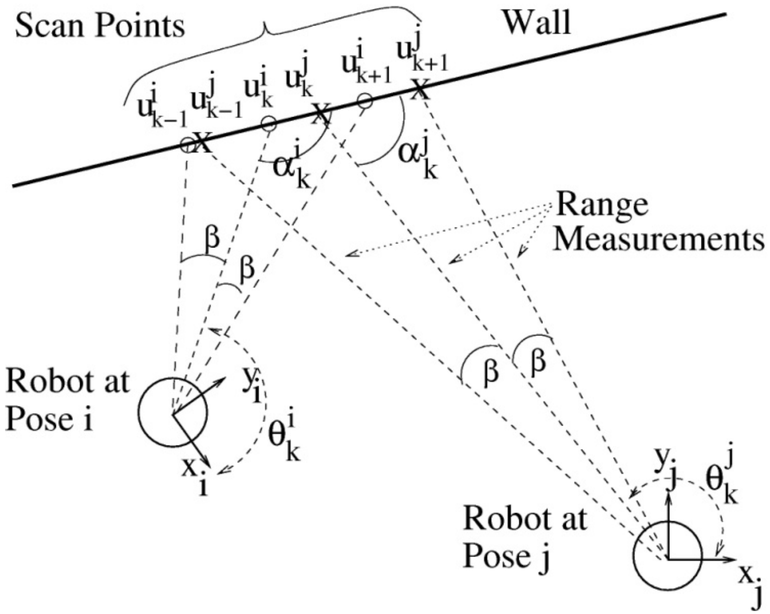
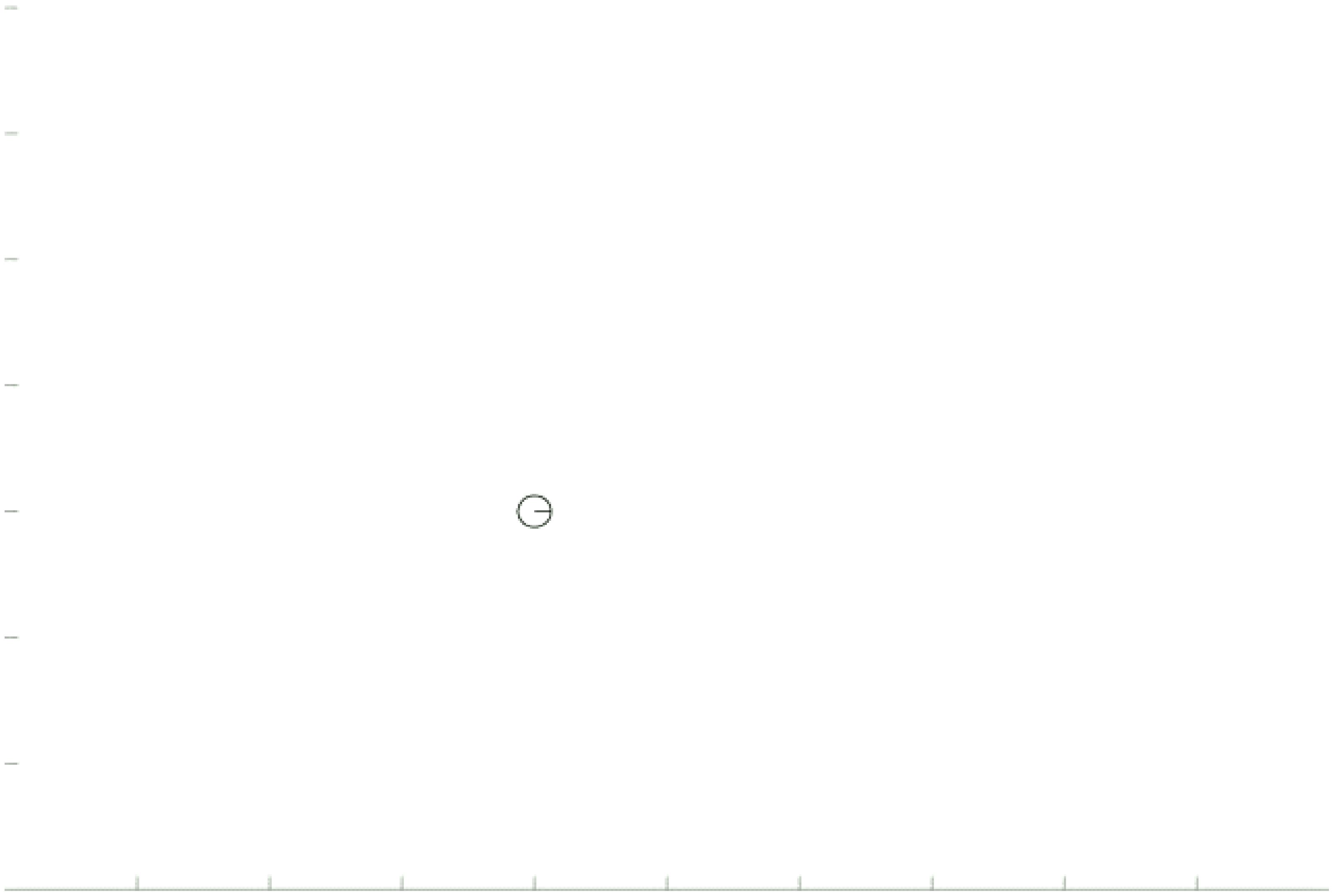
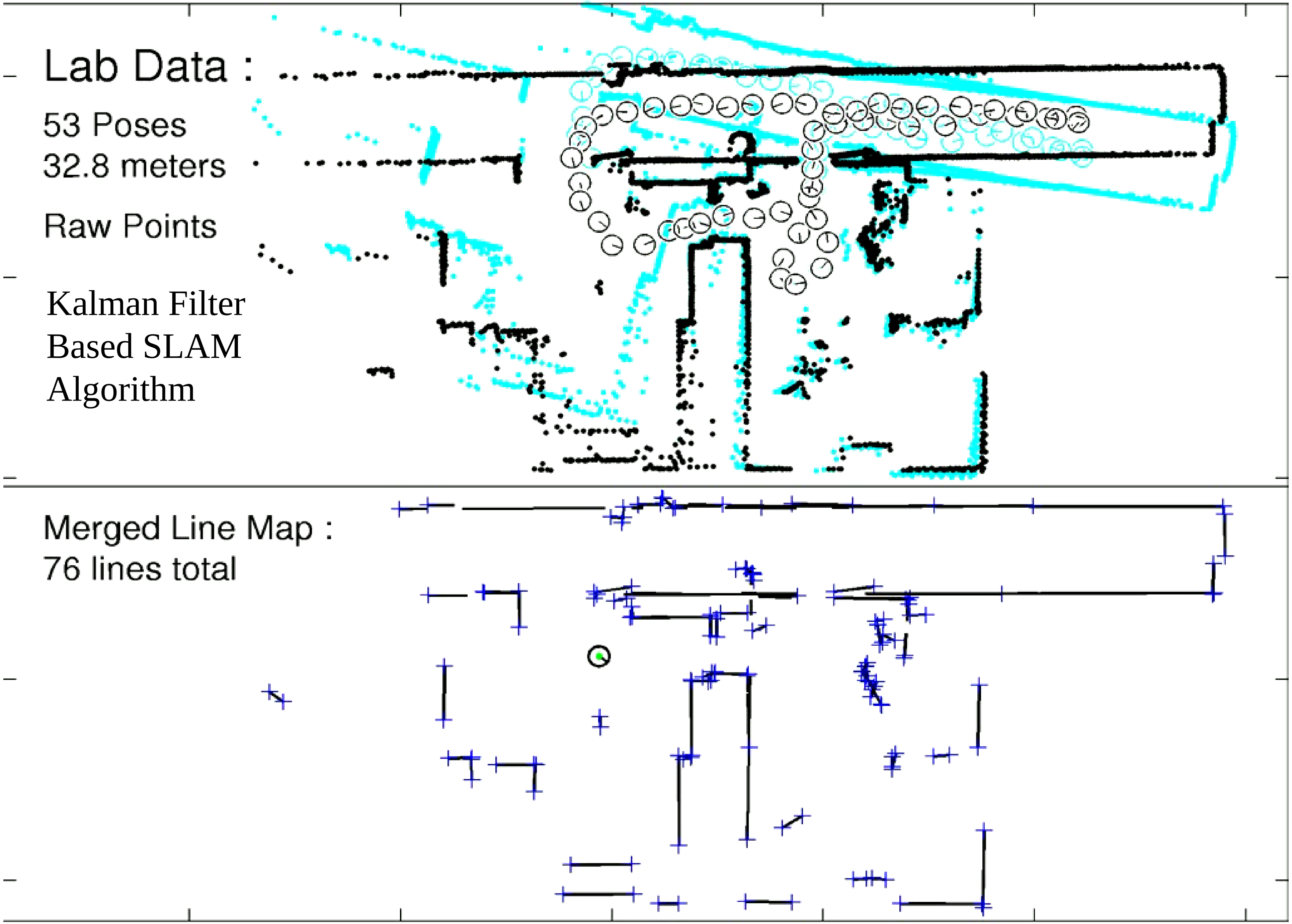
Laser Scan Matching

Range Scan Points








Lab Data :
53 Poses
32.8 meters
Raw Points
Kalman Filter
Based SLAM
Algorithm
Merged Line Map :
76 lines total

Derivation of Displacement Estimate

Let $u_{i,k}$ = location of k^{th} scan point at configuration i

$$u_{i,k} = r_k \begin{bmatrix} \cos \beta_k \\ \sin \beta_k \end{bmatrix}$$

Transform scan points in one local scan reference frame to the other reference frame which is to be matched.

- Transform points from the local frame of configuration q_{j+1} to the frame of q_j

$$u_{i+1,k}^{i} = \begin{bmatrix} \Delta x_{i+1} \\ \Delta y_{i+1} \end{bmatrix} + \begin{pmatrix} \cos \Delta\theta_{i+1} & -\sin \Delta\theta_{i+1} \\ \sin \Delta\theta_{i+1} & \cos \Delta\theta_{i+1} \end{pmatrix} u_{i+1,k}^{i+1} = T_{i+1} + R(\Delta\theta_{i+1}) u_{i+1,k}^{i+1}$$

Using an *estimate* of the displacement between frames:

$$\Delta q_{i+1} = \begin{bmatrix} \Delta x_{i+1} & \Delta y_{i+1} & \Delta\theta_{i+1} \end{bmatrix}^T = q_{i+1} - q_i$$

To *improve* the estimate of Δq_{i+1}, define an *Error Function* that models errors in the matching of associated range points in the two sets

$$E(\Delta\theta_{i+1}, T_{i+1}) = \sum_{k=1}^{N_{match}} ||u_{i+1,k}^{i} - u_{i,k}||^2 = \sum_{k=1}^{N_{match}} \left|\left| T_{i+1} + R(\Delta\theta_{i+1}) u_{i+1,k}^{i+1} - u_{i,k} \right|\right|^2$$

Derivation of Displacement Estimate

The *matching error* will be minimized when:

$$\frac{\partial E}{\partial \Delta q_{i+1}} = 0 \quad \Rightarrow \quad \frac{\partial E}{\partial T_{i+1}} = \vec{0}\,; \quad \frac{\partial E}{\partial \Delta\theta_{i+1}} = 0$$

Taking the derivatives and performing some algebra yields:

$$\begin{bmatrix} \Delta x_{i+1} \\ \Delta y_{i+1} \end{bmatrix} = \begin{bmatrix} x' - (x cos\, \Delta\theta - y sin \Delta\theta) \\ y' - (x sin \Delta\theta + y cos \Delta\theta) \end{bmatrix}\,; \quad \Delta\theta_{i+1} = \tan^{-1}(\tan \Delta\theta)$$

Where

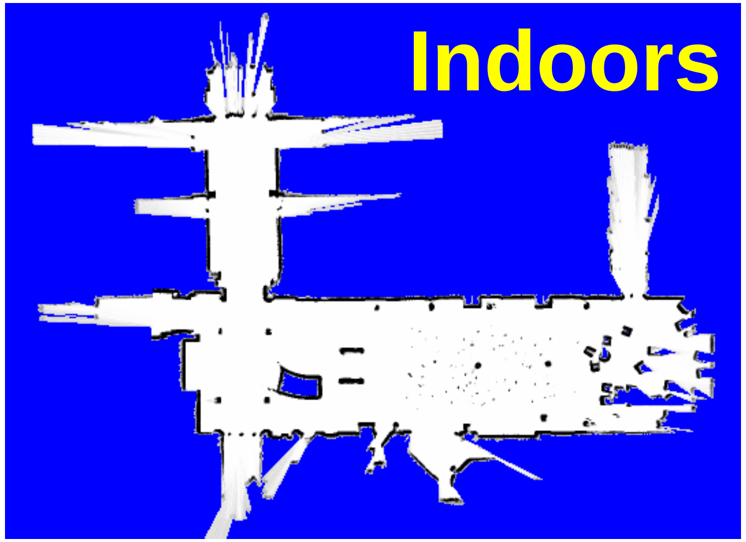
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{N_{match}} \sum_{k=1}^{N_{match}} u_{i+1,k}\,; \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{N_{match}} \sum_{k=1}^{N_{match}} u_{i,k}\,; \quad \tan \Delta\theta = \frac{\Delta_{xy'} - \Delta_{yx'}}{\Delta_{xx'} - \Delta_{yy'}}$$

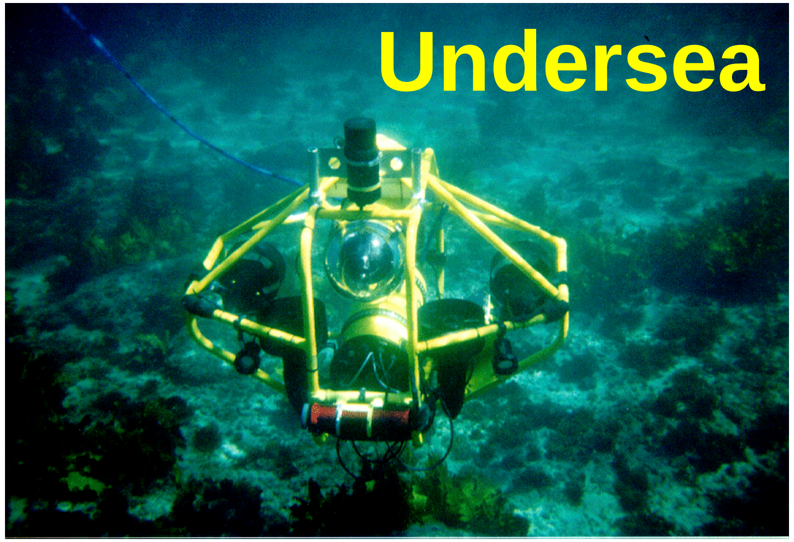
$$\Delta_{xx'} = \sum_{k=1}^{N_{match}} (x_i - x)(x_i' - x')\,; \quad \Delta_{yy'} = \sum_{k=1}^{N_{match}} (y_i - y)(y_i' - y')$$

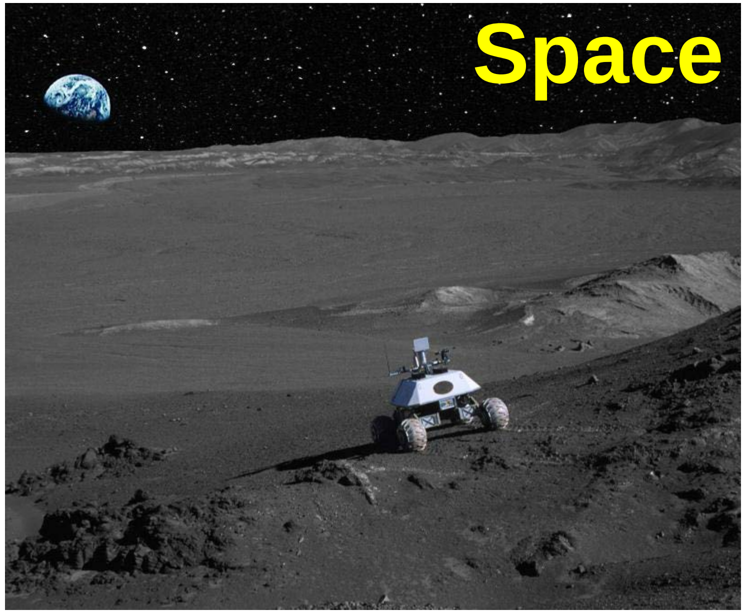
$$\Delta_{xy'} = \sum_{k=1}^{N_{match}} (x_i - x)(y_i' - y')\,; \quad \Delta_{yx'} = \sum_{k=1}^{N_{match}} (y_i - y)(x_i' - x')$$

Robot *Localization*

(where am I?)





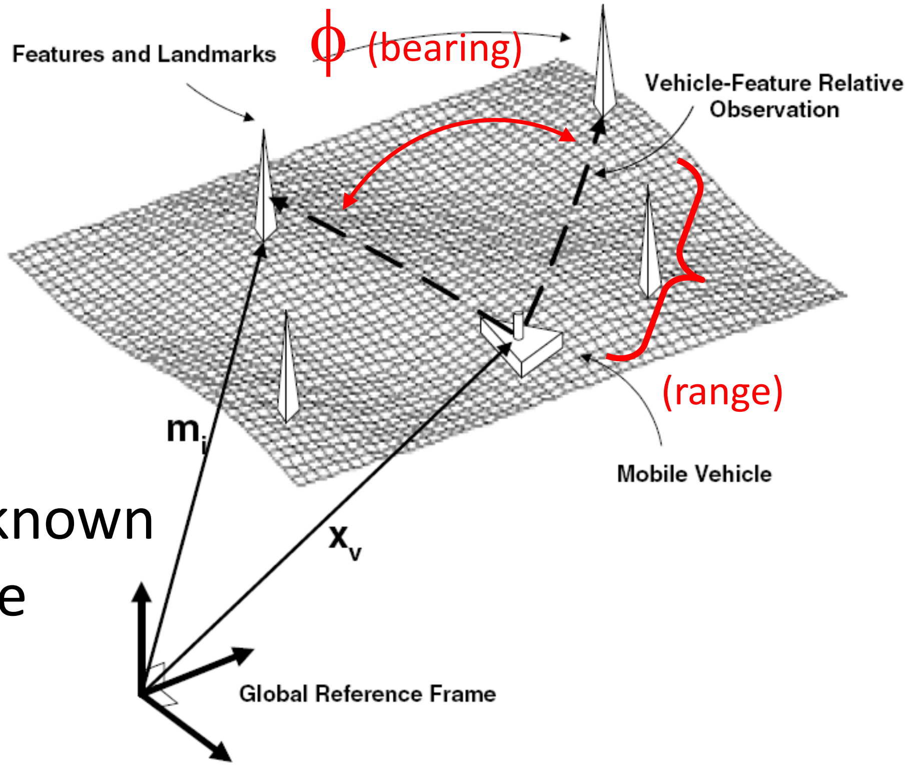


Landmark–based Localization & Mapping

Localization: A robot explores an static environment where there are known *landmarks*.

- radio beacons (Lojack)
- infrared beacons (Northstar)
- bar-code decals

Estimate the robot's position



Mapping: A robot explores an unknown static environment where there are identifiable landmarks. *E.g.:*

- doors, windows, light fixtures
- linoleum floor patterns

Build a *map* (estimate all landmark positions)

Estimation & Optimal (Kalman) Filtering

Observer

Process Dynamics: $\dot{x} = f(x,u)$ Measurement Equation: $y = h(x)$

- Given $\dot{x} = f(x,u)$ $y = h(x)$
- Calculate, infer, deduce the state x from measurements y
- E.g. the *Luenberger Observer* $\dot{x} = Ax + Bu + L(y - Cx)$

Estimator

- Given $\dot{x} = f(x,u) + \xi$ $y = h(x) + \omega$
  - ξ represents *process noise/uncertainty* (e.g., gust or unmodeled effects)
  - ω represents *measurement noise/uncertainty*
- *Estimate* (in an *optimal)* way the state x based on
  - measurements y
  - dynamic and measurement models
  - noise model(s).

Estimation Overview (continued)

Noise & Uncertainty Models for Estimation

$$\dot{x} = f(x,u) + \xi \qquad y = h(x) + \omega(t)$$

- Set-based : $\xi \in \Xi \qquad \omega \in \Omega$
- Stochastic : ξ and ω are random processes governed by $\mathrm{p}(\xi)$ and $\mathrm{p}(\omega)$

Why Estimation ?

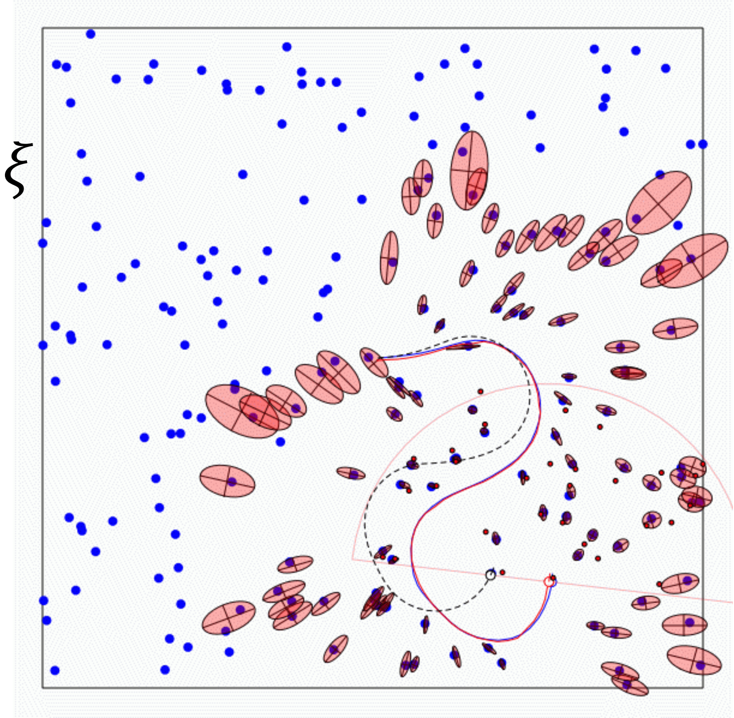
- Enables state feedback control design (separation principle)
- ***MANY*** important problems can be posed as estimation problems. *E.g.:*
  - Inertial Navigation
  - Tracking and Prediction
  - Parameter Estimation
  - Sensor Processing and Fusion

} Specialized estimation techniques & literatures

SLAM: Simultaneous Localization & Mapping

Given:

- Robot motion model: $\dot{x} = f(x,u) + \xi$
- The robot's controls, *u*
- Measurements (e.g., range, bearing) of nearby features: $y=h(x)+\omega$



Estimate:

- Map of landmarks (x_L)
- Robot's current *pose*, x_R, & its path
- Uncertainties in estimated quantities

$$\vec{x} = \begin{bmatrix} \vec{x}_R \\ \vec{x}_L \end{bmatrix} = \begin{bmatrix} \vec{x}_R \\ x_{L_1} \\ \vdots \\ x_{L_N} \end{bmatrix}$$

Robot state

Landmark positions