CDS 101/110: Midterm

(Due 5:00 pm Friday, December 9, 2016)

Instructions

- **Time Limits:** The exam is time-limited (see below for limits). The time that it takes to read these instructions is not counted against the time limit. You can take a break to eat a meal or to contact the course T.A.s or instructor. The break time does not count against your time, if you honestly do not work on the exam during the break.
  - **CDS 101 Students:** 3.0 hour time limit.
  - **CDS 110 Students:** 5.0 hour time limit.
- **Points:** The point values for each part are given so that you can budget your time. If time does not permit a complete answer, indicate how you would proceed as explicitly as possible. Please show as much of your work as possible so that you can receive the maximum partial credit.
- **Collaboration Policy:** The exam is open book. You may use the course text (Astrom and Murray 2^{nd} edition), any of the optional texts (Lewis; Friedland; Franklin, Powell and Emami-Naeni), course handouts, lecture and class notes, course problem sets and solutions, and your own handwritten notes. No other books are allowed.

  You may use a computer or calculator for carrying out numerical computations. MATLAB or Mathematica may be used. All of your answers must be hand written. You can include computer-generated plots, or hand sketch them in your solution.

  You are not allowed to use the Internet during the exam, except for accessing local computing resources, such as MATLAB, or allowed material posted on the course web site.
- **Due Date:** The exam is due by 5 p.m. Friday, December 9. It should be turned in to Gates-Thomas 250.
- **Your Solutions:** If possible, please write your solutions in a fresh exam book (blue book). We have to grade a large collections of exams in a short time and it makes things much simpler to manage if everyone uses a blue book. If you use individual sheets of paper, please number them and staple them in order. You can glue, tape, or staple any computer-generated plots into your answers.

Problem 1 (CDS 101, CDS110): (50 points)

This problem consider the design of a controller for a disk drive.

Problem #1: Figure 1 shows a highly simplified diagram of a mechanism for positioning a magnetic disk drive "read head" on the disk platter.

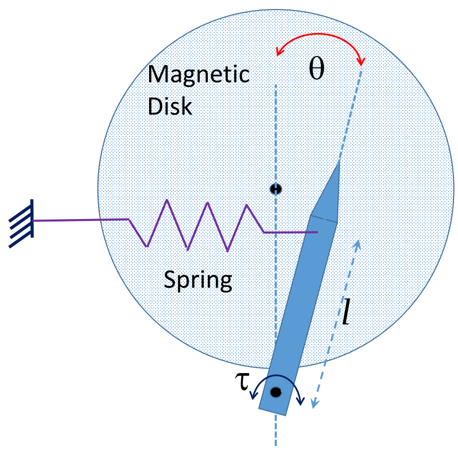

Figure 1: Schematic of Disk Drive read-head positioning mechanism.

The positioning system consists of a spring loaded arm (with arm length l) that is driven by a motor that applies a torque, τ to the arm's pivot point. This action in turn applies a force against the spring to pull the head across the magnetic platter. The dynamics governing the arm's motion is given by:

$$J\ddot{\theta} \;=\; -b\dot{\theta} \;-\; kr\sin\theta \;+\; \tau$$

where J is the rotational inertia of the arm, b is a viscous drag coefficient, which models the air drag on the arm and read head, k is the spring constant of the retraction spring, and r is the unsprung length of the spring. The angle of the arm is denoted by the angle θ. The torque on the arm generated by the motor is governed by the following equation:

$$\dot{\tau} = -a(\tau - u)$$

where u is the desired motor torque, and is the control input to the system. For the sake of practical computation below, you can assume that the key system model parameters take the following values:

$$k = \sqrt{2}, \qquad J = 100, \qquad b = 10, \qquad r = 1, \qquad l = \sqrt{2}, \qquad a = 1.0$$

Part (a): (5 points) Develop a linearized state-space description of this system, assuming a small angle approximation (on the variable θ, which is assumed to be the output variable).

Part (b): (5 points) Is this system reachable? Observable?

Part (c): (10 points) Design a state feedback controller which stabilizes the system about $\theta_{eq} = 45^o$, and sets the closed loop eignenvalues to

$$\lambda_{1,2} \ = \ -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \ .$$

Part (d): (5 points) Show that the transfer function of the linearized problem takes the form:

$$P(s) = \frac{\theta(s)}{u(s)} = \frac{a}{a+s}\left(\frac{K}{s^2 + 2\zeta\omega + \omega^2}\right).$$

Part (e): (10 points) Sketch the Bode Plot for the linearized open loop system. Even if you use MATLAB to plot the exact bode plot, please diagram the approximate Bode plots using the Bode plot sketching techniques.

Part (f): (15 points) Design a compensator that provides tracking with less than 10% error up to 1 rad/sec and has at least 60^o phase margin?

Problem 2 (CDS 101 & CDS 110: (45 points)

Consider the magnetic levitation system that we discussed in class. Recall that linearized version of its governing dynamics lead to a transfer function

$$P(s) = \frac{k}{s^2 - r^2}$$

where $k = 4000$ and $r = 25$, and the input is the current to the magnetic coil, and the output is the intensity of an infrared sensor.

Part (a): (10 points) Design a PID controller which stabilizes this system.

Part (b): (10 points) Sketch the bode plot for the loop transfer function (including your PID controller)

Part (c): (10 points) Plot the Nyquist plot of the loop transfer function (including you PID controller), and confirm using the Nyquist criterion that your closed loop system is stable.

Part (d): (5 points) Derive the equivalent state-space representation of this system.

Part (e): (10 points) Design a state feedback controller which places the poles of the closed loop system system at $p_{1,2} = -20 \pm 30i$.

Problem 3 (CDS 110): (50 points) Consider again the problem of designing a cruise control for a car. The following transfer function represents the vehicle and engine dynamics

$$P(s) = \frac{Tba/m}{(s+a)(s+c/m)}$$

where $b = 25$ is the transmission gain, $T = 200$ is the conversion factor between throttle and steady state motor torque, $a = 0.2$ is the engine lag coefficient, $m = 1000kg$ is the vehicle mass, and $c = 50Nsec/m$ is the air drag coefficient.

Part (a): (15 points) Consider a proportional controller for the car: $u = k_p(r - y)$, where y is the measured vehicle speed, and r is the desired vehicle speed. Within the context of negative feedback, this gives

$$C(s) = k_p.$$

Assuming that $k_p = 0.1$, compute the steady state error, gain, phase margin, rise time, overshoot, and closed loop poles of this system.

Part (b): (15 points) Now consider a proportional and integral controller:

$$C(s) = k_p + \frac{k_i}{s}.$$

For $k_p = 0.05$ and $k_i = 0.001$:

- determine if the closed loop system is stable.
- calculate the gain and phase margin
- estimate the steady state error and the percent overhshoot
- estimate the bandwidth of the closed loop system.

Part (c): (20 points) Let us now consider what happens if we add a small amount of delay in the feedback loop. A pure time delay of τ seconds satisfies the equation (where u is the input to the delay block, and z is the output of the delay)

$$z(t) = u(t - \tau).$$

The transfer function of a delay can be expressed exactly as

$$G(s) = e^{-s\tau}.$$

We can use the "Padé approximation" to represent the effect of a delay as a LTI transfer function. Up to second order, the Padé approximation for delay τ takes the form:

$$G(s) = \frac{1 - \tau s/2 + (\tau s)^2/12}{1 + \tau s/2 + (\tau s)^2/12}.$$

Assume that there is a delay of τ seconds between the output of the plant (the speed) and the input. For the PI controller of Part (b), with $k_p = 0.05$ and $k_i = 0.001$ (the same values as Part (b)), compute the gain and phase margin, as well as the overshoot, for two cases: $\tau = 0.25$ seconds, and $\tau = 0.75$ seconds.

Problem 4 (CDS 110): (50 points)

Consider a system with transfer function

$$P(s) = \frac{a}{(s+1)^2(a-s)} \qquad 0.05 < a < 0.5$$

with a simple proportional feedback controller: $C(s) = k_p$.

Part (a): (10 points) Sketch the Bode plot for the loop transfer function, indicating key features in magnitude and frequency.

Part (b): (10 points) Sketch the Nyquist plot for the loop transfer function, and determine if the closed loop system is stable for $k_p = 2$ and $k_p = -2$.

Part (c): (15 points) Determine the range of values of k_p which stabilize the system.