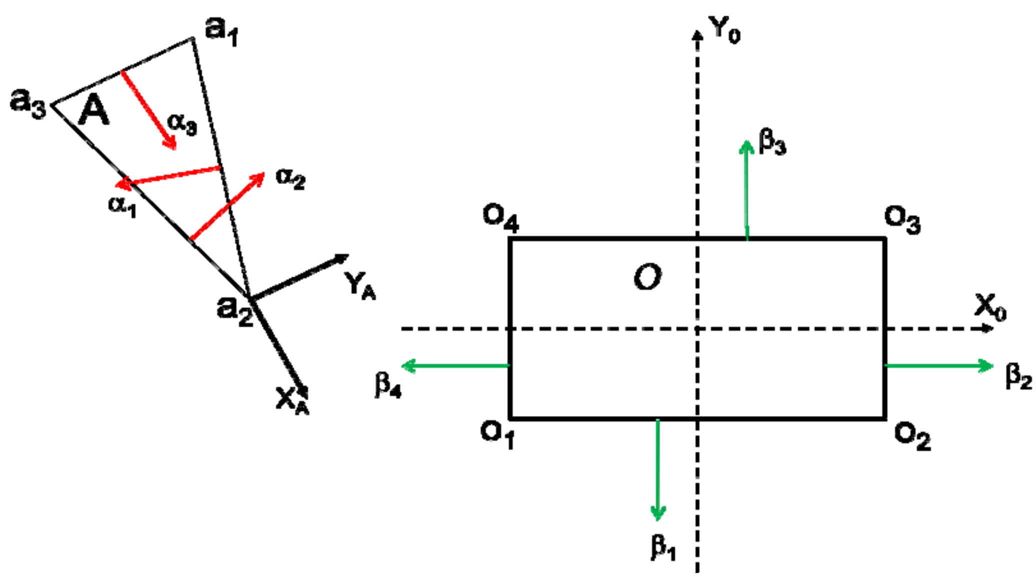
The "Star" Algorithm for
Computing Fixed-Orientation C-Obstacle Slices of Convex Objects

(ME/CS 132)

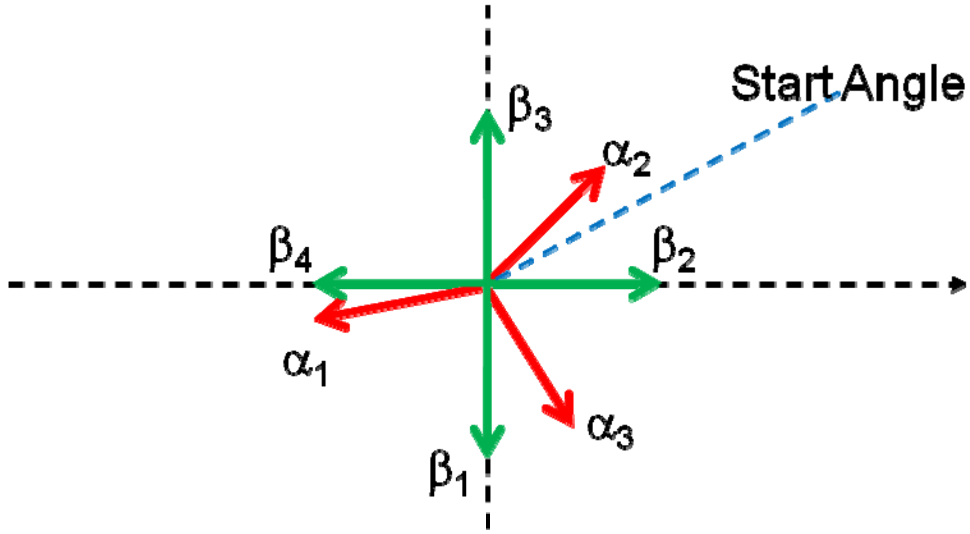
The *star algorithm* is based on a simple geometric construction. Let $A(q)$ the set of points comprising the robot when it lies at configuration q. Let O denote an obstacle, and let CO denote its associated configuration space obstacle. If the robot's orientation is fixed at some angle, θ^*, the boundary of CO in this fixed orientation slice through configuration space corresponds to all physical configurations where A is touching O. Practically, this boundary can be constructed by "sliding" the robot (in a fixed orientation θ^*) all around O while maintaining contact between A and O. The path traced out be the origin of A's body fixed frame is exactly the shape of the fixed-orientation c-obstacle slice.

While this construction is clear from a graphical point of view, one needs a systematic procedure in order to turn this concept into an implementable algorithm. The key to a straightforward implementation is the *star diagram*, which is now defined.

Let the vertices of A be labeled successively $a_1, a_2, \ldots, a_{NA}$ where NA is the number of A's vertices. Similarly, let O's vertices be labeled $o_1, o_2, \ldots, o_{NO}$. Let E^A_i denote the i^{th} edge of A, which connects a_i to a_{i+1}. Similarly, let E^O_i denote the i^{th} edge of O. Let α_i denote the ***inward pointing normal*** to E^A_i, while the ***outward pointing normal*** to E^O_i is denote β_i. These variables are depicted in the figure below.



The ***star diagram*** is constructed from a translation of all of the normal vectors to a common origin. For example, the star diagram of the figure above is shown below. It is the angle of the vectors (say with respect to the x-axis) that matters in this diagram.



The star diagram assists you in determining the type of sliding contact that holds around the perimeter of the fixed-orientation slice, and the sequence of sliding operations that will construct the c-obstacle boundary in a continuous fashion. To start the construction, select a *start angle.* Typically the start angle will be zero degrees. Next choose either a clockwise or counterclockwise direction to proceed around the star diagram. Either choice will work fine, as long as you consistently move in the chosen direction. In our example, the start angle is shown by the blue dashed line, and we will proceed in a clockwise fashion. Consequently, the c-obstacle boundary will be constructed in a clockwise fashion.

The first normal vector, n_1, reached by proceeding in the chosen direction around the star diagram, (normal vector β_2 in our example) defines the first segment of the c-obstacle boundary to be constructed. If n_1 is associated with the obstacle (as is the case in our example), then the first segment will be constructed by sliding a vertex of A along the associated edge of *O*. If n_1 was derived from the robot, then the segment will be constructed by sliding the associated edge of *A* along a vertex of *O.* The vertex can be determined as follows. Find the two closest normals from the opposite object (i.e., if n_1 is associated with O, then these normals must be associated with *A,* and vice-versa) that bound n_1 in the clockwise and counterclockwise directions. Let these normal vectors be denoted n_j and n_k. In our example, normal vectors α_2 and α_3 are closest to β_2, and lie on either side of β_2. The edge associated with n_1 contacts the vertex that joins the edges associated with normals n_j and n_k. In our example, vertex a_3 joins edges E^{A}_{2} and E^{A}_{3}. Thus this segment physically corresponds to the sliding of vertex a_3 along E^{O}_{2}.

The construction of the fixed-orientation c-obstacle boundary then continues by proceeding around the diagram. In our example, the next normal vector reached by continuing in a clockwise direction is α_3, which is bounded by normal vectors β_1 and β_2. Thus, the next contiguous segment of the c-obstacle boundary will physically correspond to sliding of E^{A}_{3} (which is associated with α_3) along vertex o_2 (the vertex joining edges E^{O}_{1} and E^{O}_{2}, which are the associated adjacent normal vectors β_1 and β_2). Once all of the normal vectors in the star diagram have been processed, the boundary of the fixed-orientation c-obstacle "slice" is complete.